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Sensitivity of the theta projection technique to the functional form of the theta interpolation/extrapolation function

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This paper estimates various interpolation/extrapolation functions for the theta parameters within the theta projection technique and assesses their ability to accurately predict the creep life of 1CrMoV rotor steel. The theta interpolation/extrapolation functions were estimated using short-term data collected at the Interdisciplinary Research Laboratories in Swansea and the accuracy of the predictions assessed using longer-term rupture times published by the National Research Institute of Metals in Japan. It was found that the theta interpolation/extrapolation function traditionally used by theta practioners was not the best predictor of long term life and that the most accurate long term life predictions were obtained using simpler functional forms. Further, it was found that more accurate lifetime predictions are obtained by estimating such interpolation/extrapolation functions using weighted rather than ordinary least squares. © *2002 Kluwer Academic Publishers*

1. Introduction

When designing materials for high temperature service the design criteria for long-term operation must guarantee that creep failure should not occur within the planned service life. Such creep fracture represents an obvious life limiting design consideration, as fracture of the pipe work or turbine blades used by the nuclear powered electricity generating plants would prove catastrophic. For this reason time to rupture has been a major criterion used to assess the accuracy of a creep property projection technique. The theta projection technique is one of a number of models currently available.

The theta projection technique has already been shown to yield excellent medium-term predictions for both the time to rupture and the minimum creep rate of 1CrMoV rotor steels (Evans [1] and Evans *et al.* [2]). The technique has the added advantage over traditional parametric procedures (such as the Larson-Miller technique [3]) in that creep property predictions are not limited to the rupture time and the mathematical equations have very recently been given some sound theoretical footings [4]. Times to various strains can also be predicted using the theta projection technique. Further, the technique has recently been shown, by Wang and Evans [5], to outperform both the Continuum Damage model of Othman and Hayhurst [6] and the CRISPEN model of Ion *et al.* [7] in the prediction of long term rupture times and minimum creep rates for 316 stainless steel.

Since its first appearance in the literature during the mid 1980's, the theta projection technique has undergone a number of modifications. On the theoretical side Evans [4] has given the creep curve equation used by the theta projection technique a theoretical explanation. On the practical side, this creep curve equation has been modified to improve the fit to the experimental data at very small strains [8, 9]. This has involved the introduction of two additional parameters into the creep curve equation. Most recently an improved method for deriving the weights required for a weighted least squares estimate of the theta interpolation/extrapolation function has been proposed and applied [10].

However, little research has gone into validating the functional form of the theta interpolation/extrapolation function. This function is used within the theta projection technique to project the creep curve parameters obtained at accelerated test conditions (and thus the creep curve and rupture time) to design stresses and temperatures. Yet it is this stage of the theta projection technique that is so crucial to reliable predictions of long term creep life. The purpose of this paper is to compare the theta interpolation/extrapolation function that is traditionally used within the theta projection technique with other possible functional forms suggested by various theories of creep and rate processes. Such a comparison will be made in terms of the accuracy of the resulting long term rupture time predictions.

To achieve this aim this paper is structured as follows. In the next section the experimental procedures and databases used are reviewed. The following section outlines the theta projection technique, how it is traditionally applied, what the plausible functional forms are for the theta interpolation/extrapolation function and how the accuracy of any set of rupture time predictions can be measured. Then in Section 4, these different interpolation/extrapolation functions are estimated using both ordinary and weighted least squares and the impact of these different theta interpolation/extrapolation functions on the accuracy of long term creep life predictions is then assessed. This will be achieved using two sources of data on 1CrMoV rotor steel. A final section concludes.

2. Experimental procedures

Two sources of data are used in this present investigation. The first source of data comes from testing a batch of material representing the lower bound creep strength properties anticipated for 1CrMoV rotor steels. This batch of material was tested at the IRC creep laboratories (Swansea) as part of a consortium testing program using high precision constant stress machines. The

TABLE I Composition and heat treatment for 1CrMoV rotor steel

	Chemical composition (wt%)												
Batch[a]	C	Si	Mn	S	P	Cr	**Ni**	Mo	V	Cu	Al	Sn	Heat treatment
IRC	0.26	0.23	0.64	0.002	0.006	0.98	**0.64**	0.74	0.31	0.04	0.007	0.004	Normalized 1243 K; Tempered 973 K
VaA	0.28	0.2	0.72	0.001	0.015	1.02	**0.32**	1.12	0.27	0.2	0.002	0.008	Normalized 1233 K; Tempered 933 K
VaB	0.28	0.18	0.75	0.009	0.012	1.00	**0.32**	1.25	0.27	0.14	0.002	0.009	Normalized 1243 K; Tempered 933 K
VaC	0.29	0.2	0.75	0.009	0.01	1.00	**0.34**	1.25	0.26	0.14	0.002	0.008	Normalized 1233 K; Tempered 938 K
VaD	0.30	0.28	0.72	0.006	0.014	0.93	**0.35**	1.22	0.21	0.16	0.002	0.009	Normalized 1238 K; Tempered 918 K
VaE	0.30	0.26	0.79	0.015	0.016	1.03	**0.32**	1.13	0.23	0.19	0.002	0.009	Normalized 1238 K; Tempered 920 K
VaG	0.29	0.26	0.76	0.007	0.009	1.12	**0.45**	1.18	0.23	0.07	0.003	0.01	Normalized 1233 K; Tempered 920 K
VaH	0.29	0.26	0.77	0.007	0.009	1.12	**0.46**	1.2	0.23	0.08	0.004	0.01	Normalized 1248 K; Tempered 930 K
VaJ	0.29	0.21	0.66	0.008	0.01	1.07	**0.51**	1.29	0.23	0.06	0.005	0.01	Normalized 1248 K; Tempered 925 K
VaR	0.3	0.27	0.70	0.012	0.012	1.1	**0.44**	1.35	0.27	0.11	0.003	0.008	Normalized 1248 K; Tempered 918 K

[a]IRC is the batch of material tested at the IRC (Swansea) as part of a consortium group and VaA to VaR are the nine batches of steel tested by NRIM. The balance for all the above batches is Fe.

TABLE II Number of specimens tested at accelerated and long-term test conditions

	Number of specimens[a] tested at the following temperatures								
Stress (MPa)	723 K	783 K	803 K	823 K	843 K	863 K	898 K	923 K	948 K
425		1							
415		1							
412	**1**	**1**							
400		1							
380		1	1						
373	**1**	**9**							
370			1						
360		1	1						
350				1					
345		1							
340			1	1	1				
333		**9**	**1**	**2**					
330				1	1				
320			1						
314		**2**							
310				1	1	1			
300			1						
294		**9**		**9**					
290				1	1	1			
270				1	1	2			
265		**9**	**1**	**9**	**1**				
250				1	1				
240						1			
235		**9**	**1**	**9**		**9**			
230						1			
220						1			
216			**1**		**1**				
196				**9**	**1**				
177				**9**	**1**	**9**			
157				**9**	**1**				
137				**9**	**1**	**9**	**1**	**9**	
118					**1**				
98						**9**	**1**	**9**	**1**
69						**9**	**1**	**9**	
61						**9**			
47						**6**		**9**	**1**

[a]Normal numerals correspond to the accelerated tests carried out at the IRC laboratories (Swansea) on a single batch of material. Bold numerals correspond to the longer-term tests carried out by NRIM on nine batches of material. In addition to the above tests a further single specimen was tested at the IRC laboratories at 833 K and 270 MPa.

material consisted of fabricated header pipe with an outside diameter of 400 mm and a wall thickness of 60 mm. The chemical composition of this batch of material and the heat treatment received is shown in the top row of Table I. Following this heat treatment the material had, at room temperature, a tensile strength of 763 MPa, an elongation of 19% and a 0.2% proof stress of 634 MPa.

The test matrix is shown in Table II using normal numerals and it can be seen that the 33 specimens were all tested at accelerated stresses and temperatures that therefore lead to relatively short times to rupture. For all specimens some 400 creep strain/time readings were taken and normal creep curves were observed under all the test conditions. As complete creep curves at various

TABLE III Least squares parameter estimates for various interpolation/extrapolation functions for Θ_1

	Variable						
Model for Θ_1 Model for $W_1^*\Theta_1$	Constant W_1	T W_1^*T	$1/T$ W_1/T	τ/T $(W_1^*\tau)/T$	τ^*T $W_1^*\tau^*T$	Ln[$1/T$] W_1^*Ln[$1/T$]	τ $W_1^*\tau$
Equation 2c	−11.4167	0.0072	–	–	5.2687E-06	–	−0.0015
	(−0.55)	(0.29)			(0.07)		(−0.02)
	−30.2146	0.0277	–	–	−3.3570E-06	–	0.0351
	(−3.73)*	(2.86)*			(−1.08)		(1.38)
Equation 4a	2.5443	–	−6611.67	2.3038	–	–	–
	(0.51)		(−1.38)	(0.88)			
	9.8384	–	−14284.6	6.5632	–	–	–
	(3.50)*		(−5.34)*	(4.85)*			
Equation 4b	1.1805	–	−5491.34	−1.2202	–	–	0.0043
	(0.06)		(−0.32)	(−0.02)			(0.07)
	17.9866	–	−20927.26	29.2937	–	–	−0.0280
	(2.24)*		(−3.13)*	(1.39)			(−1.08)
Equation 4d	−1135.0352	–	115171.87	1.9247	–	−147.426	–
	(−0.70)		(0.66)	(0.71)		(−0.70)	
	−450.092	–	35166.37	6.2553	–	−59.5745	–
	(−0.62)		(0.45)	(4.32)*		(−0.64)	
Equation 4e	−3330.7844	–	373930.36	−84.4887	–	−427.689	0.1048
	(−1.24)		(1.22)	(−1.00)		(−1.24)	(1.03)
	956.9716	–	−127989.2	53.9764	–	120.5089	−0.0577
	(0.63)		(−0.74)	(1.20)		(0.62)	(−1.06)

Values in parenthesis are student t values. The first row for each equation contains the coefficient in front of each variable shown at the top of the table as estimated by ordinary least squares and the second row contains the same coefficients as estimated using weighted least squares. W_1 is the weighting for the estimate made of Θ_1, τ is stress, T is temperature and Ln is the natural log. *Indicates a statistically significant variable at the 5% significance level.

TABLE IV Least squares parameter estimates for various interpolation/extrapolation functions for Θ_2

	Variable						
Model for Θ_2 Model for $W_2^*\Theta_2$	Constant W_2	T W_2^*T	$1/T$ W_2/T	τ/T $(W_2^*\tau)/T$	τ^*T $W_2^*\tau^*T$	Ln[$1/T$] W_2^*Ln[$1/T$]	τ $W_2^*\tau$
Equation 2c	−99.1996	0.0918	–	–	−6.8200E-05	–	0.0935
	(−2.74)*	(2.11)*			(−0.51)		(−0.86)
	−15.1599	−0.0066	–	–	0.0001833	–	−0.1207
	(−0.56)	(−0.20)			(1.87)		(−1.50)
Equation 4a	46.7885	–	−58076.1	31.2317	–	–	–
	(5.40)*		(−6.97)*	(6.85)*			
	32.5569	–	−44095.3	25.0847	–	–	–
	(4.44)*		(−6.22)*	(6.59)*			
Equation 4b	62.1545	–	−70698.8	70.9362	–	–	−0.0486
	(1.75)		(−2.40)*	(0.80)			(−0.45)
	−18.1458	–	−2194.51	−104.5750	–	–	0.1577
	(−0.67)		(−0.10)	(−1.56)			(1.94)
Equation 4d	1411.66	–	−204192.0	31.6864	–	176.8820	–
	(0.50)		(−0.67)	(6.73)*		(0.48)	
	3078.0400	–	−371904.7	26.8232	–	394.4021	–
	(1.36)		(−1.52)	(6.75)*		(1.34)	
Equation 4e	6492.03	–	−802892.0	231.624	–	825.3370	−0.2425
	(1.41)		(-1.53)	(1.60)		(1.39)	(−1.38)
	−1876.1500	–	211021.6	−152.6745	–	−238.236	0.2149
	(−0.45)		(0.44)	(−1.20)		(−0.45)	(1.41)

Values in parenthesis are student t values. The first row for each equation contains the coefficient in front of each variable shown at the top of the table as estimated by ordinary least squares and the second row contains the same coefficients as estimated using weighted least squares. W_2 is the weighting for the estimate made of Θ_2, τ is stress, T is temperature and Ln is the natural log. *Indicates a statistically significant variable at the 5% significance level.

test conditions are available, the theta interpolation/extrapolation functions will be estimated from just this set of data.

To assess the impact of different functional forms of the theta interpolation/extrapolation function on rupture time predictions made using the theta projection technique an additional source of data was also used. This source of data has much longer rupture lives than the one above and is made up of nine different batches of material. All these batches were tested at the National Research Institute for Metals in Japan (NRIM). The forgings used in this test program were sampled at random from a commercial stock and the test specimens had 10 mm diameters and 50 mm gauge lengths. The creep rupture tests were constant stress tests, conducted as specified in JIS Z 2272 [11], and published in

TABLE V Least squares parameter estimates for various interpolation/extrapolation functions for Θ_3

	Variable						
Model for Θ_3 Model for $W_3^*\Theta_3$	Constant W_3	T W_3^*T	$1/T$ W_3/T	τ/T $(W_3^*\tau)/T$	τ^*T $W_3^*\tau^*T$	$\text{Ln}[1/T]$ $W_3^*\text{Ln}[1/T]$	τ $W_3^*\tau$
Equation 2c	65.0119	−0.0826	–	–	0.0002816	–	−0.2413
	(1.61)	(−1.70)			(1.90)		(−1.98)*
	95.1950	−0.1138	–	–	0.0003442	–	−0.30417
	(2.57)*	(−2.53)*			(2.55)*		(−2.75)*
Equation 4a	−0.4022	–	−2093.90	−8.6104	–	–	–
	(−0.04)		(−0.21)	(−1.59)			
	−5.9728	–	6440.54	−18.5609	–	–	–
	(−0.98)		(1.07)	(−4.65)*			
Equation 4b	−67.1698	–	52753.6	−181.133	–	–	0.2109
	(−1.67)		(1.58)	(−1.80)			(1.71)
	−88.5745	–	74083.80	−223.9300	–	–	0.2515
	(−2.36)*		(2.39)*	(−2.42)*			(2.22)*
Equation 4d	6681.71	–	−717448.0	−6.3839	–	865.979	–
	(2.14)*		(−2.14)*	(−1.23)		(2.14)*	
	5435.4200	–	−576735.0	−15.0440	–	705.1790	–
	(2.47)*		(−2.45)*	(−3.80)*		(2.47)*	
Equation 4e	6257.7800	–	−667489.0	−23.0677	–	811.8680	0.0202
	(1.19)		(−1.11)	(−0.14)		(1.20)	(0.10)
	3803.8860	–	−375174.0	−109.3560	–	498.6023	0.1142
	(1.20)		(−1.02)	(−0.84)		(1.23)	(0.72)

Values in parenthesis are student t values. The first row for each equation contains the coefficient in front of each variable shown at the top of the table as estimated by ordinary least squares and the second row contains the same coefficients as estimated using weighted least squares. W_3 is the weighting for the estimate made of Θ_3, τ is stress, T is temperature and Ln is the natural log. *Indicates a statistically significant variable at the 5% significance level.

TABLE VI Least squares parameter estimates for various interpolation/extrapolation functions for Θ_4

	Variable						
Model for Θ_4 Model for $W_4^*\Theta_4$	Constant W_4	T W_4^*T	$1/T$ W_4/T	τ/T $(W_4^*\tau)/T$	τ^*T $W_4^*\tau^*T$	$\text{Ln}[1/T]$ $W_4^*\text{Ln}[1/T]$	τ $W_4^*\tau$
Equation 2c	−96.1948	0.0813	–	–	8.4499E-07	–	0.0513
	(−5.46)*	(3.84)*			(0.01)		(0.97)
	−105.0890	0.0923	–	–	−1.8273E-05	–	0.0664
	(−8.25)*	(5.91)*			(−0.39)		(1.75)
Equation 4a	54.6331	–	−69076.1	42.8071	–	–	–
	(12.6)*		(−16.50)*	(18.70)*			
	58.7890	–	−72203.90	42.0042	–	–	–
	(20.8)*		(−25.2)*	(22.9)*			
Equation 4b	47.0780	–	−62869.8	23.2854	–	–	0.0239
	(2.64)*		(−4.25)*	(0.52)			(0.44)
	58.2955	–	−71807.80	40.7772	–	–	0.0015
	(4.24)*		(−6.42)*	(1.22)			(0.04)
Equation 4d	−703.058	–	12038.5	42.5547	–	−98.1940	–
	(−0.49)		(0.08)	(18.0)*		(−0.53)	
	−1439.4200	–	88165.70	41.1385	–	−194.188	–
	(−1.64)		(0.94)	(22.3)*		(−1.71)	
Equation 4e	−3349.6500	–	323926.00	−61.6014	–	−436.002	0.1263
	(−1.45)		(1.23)	(−0.85)		(−1.47)	(1.44)
	−3177.9394	–	298225.57	−45.5069	–	−415.058	0.1070
	(−2.59)*		(2.12)*	(−1.02)		(−2.64)*	(1.94)

Values in parenthesis are student t values. The first row for each equation contains the coefficient in front of each variable shown at the top of the table as estimated by ordinary least squares and the second row contains the same coefficients as estimated using weighted least squares. W_4 is the weighting for the estimate made of Θ_4, τ is stress, T is temperature and Ln is the natural log. *Indicates a statistically significant variable at the 5% significance level.

their No. 9B data sheet [12]. This NRIM data set only contains times to rupture and minimum creep rates (the only recorded strains were those at rupture) and the full test matrix is shown in Table II using bold numerals. The chemical composition of each batch together with their heat treatments is summarised in Table I.

It can be seen from Table I that these two sources of data are broadly similar in composition and they received similar heat treatments—with one notable exception. The batch of material tested at the IRC has roughly twice as much nickel. This can be expected to lead to slightly lower rupture times in the NRIM data set when compared to the IRC data set at equivalent stresses and temperatures. This expectation is confirmed in Section 4 below.

3. The θ projection technique

3.1. The general model

The θ projection technique has a number of basic steps. First, there is the experimental stage where uniaxial (or multiaxial) constant stress creep curves are measured over a narrow range of accelerated stresses and temperatures. Second, the form of these creep curves are modelled in such a way as to give a good description of the experimental data. A single creep curve at steady uniaxial stress τ and absolute temperature T can be modelled using a general functional form

$$\varepsilon = \eta(t, \Theta_1, \Theta_2, \ldots, \Theta_j, \ldots, \Theta_m), \tag{1a}$$

where η is some non-linear function, ε is the uniaxial creep strain at time t and Θ_j are numerical parameters that can be determined from the experimental creep curves using a suitable estimation technique.

The third stage of the theta projection technique is to project creep curves to stresses and temperatures—either within (i.e., interpolation) or outside (i.e., extrapolation) the original range of accelerated test conditions using a suitable theta interpolation/extrapolation function

$$\Theta_j = g_j(\tau, T, b_{j1}, b_{j2}, \ldots, b_{jk}, \ldots, b_{jp}). \tag{1b}$$

Lastly, the required creep properties (such as the minimum creep rate or time to $x\%$ strain) can be 'read off' these projected creep curves. This, broadly speaking is how the theta projection technique works.

TABLE VII Mean absolute percentage prediction errors made using the 4Θ prediction technique with the interpolation/extrapolation function given by Equation 2c

Temperature	Type of prediction	Unweighted (%)	Weighted (%)
783 K	Interpolation	14.63	12.54
	Extrapolation	97.4	68.82
	Overall	88.2	62.57
803 K	Interpolation	35	36.42
	Extrapolation	148.63	60.49
	Overall	80.45	46.05
823 K	Interpolation	33.01	24.81
	Extrapolation	420.25	41.74
	Overall	382.61	40.09
843 K	Interpolation	12.97	38.18
	Extrapolation	1014.97	149.49
	Overall	552.51	98.12
863 K	Interpolation	36.26	38.00
	Extrapolation	3379.59	482.32
	Overall	3015.48	435.45
923 K	Extrapolation	188.51	290.19
Other	Extrapolation	417.29	389.64
	Overall	371.39	343.08
All temperatures	Interpolation	27.59	29.68
	Extrapolation	1110.9	217.7
	Overall	**973.4**	**193.84**

The interpolation error is the mean absolute % prediction error made by the theta prediction technique using the interpolation/extrapolation function given by Equation 2c at the accelerated test conditions shown in Table II. The extrapolation error is the mean absolute % prediction error made by the theta prediction technique using the interpolation/extrapolation function given by Equation 2c at the NRIM test conditions shown in Table II. The overall error is the mean absolute % prediction error made by the theta prediction technique using the interpolation/extrapolation function given by Equation 2c at all the test conditions shown in Table II. Other temperatures are 723 K, 803 K, 898 K and 948 K. Unweighted refers to prediction errors made using the ordinary least squares estimates of the coefficients of Equation 2c shown in Tables III–VI. Weighted refers to prediction errors made using the weighted least squares estimates of the coefficients of Equation 2c shown in Tables III–VI.

TABLE VIII Mean absolute percentage prediction errors made using the 4Θ prediction technique with the interpolation/extrapolation function given by Equation 4a

Temperature	Type of prediction	Unweighted (%)	Weighted (%)
783 K	Interpolation	21.29	16.31
	Extrapolation	82.47	76.29
	Overall	164.56	69.63
803 K	Interpolation	35.62	35.56
	Extrapolation	226.77	43.38
	Overall	112.08	38.69
823 K	Interpolation	37.57	25.67
	Extrapolation	454.53	43.31
	Overall	414	41.60
843 K	Interpolation	15.26	42.30
	Extrapolation	687.98	52.71
	Overall	377.49	47.91
863 K	Interpolation	31.37	52.73
	Extrapolation	1180.65	39.21
	Overall	1057.34	40.19
923 K	Extrapolation	166.30	82.69
Other	Extrapolation	363.28	341.86
	Overall	324.24	302.55
All temperatures	Interpolation	29.29	34.58
	Extrapolation	543.59	64.94
	Overall	**478.31**	**61.09**

The interpolation error is the mean absolute % prediction error made by the theta prediction technique using the interpolation/extrapolation function given by Equation 4a at the accelerated test conditions shown in Table II. The extrapolation error is the mean absolute % prediction error made by the theta prediction technique using the interpolation/extrapolation function given by Equation 4a at the NRIM test conditions shown in Table II. The overall error is the mean absolute % prediction error made by the theta prediction technique using the interpolation/extrapolation function given by Equation 4a at all the test conditions shown in Table II. Other temperatures are 723 K, 803 K, 898 K and 948 K. Unweighted refers to prediction errors made using the ordinary least squares estimates of the coefficients of Equation 4a shown in Tables III–VI. Weighted refers to prediction errors made using the weighted least squares estimates of the coefficients of Equation 4a shown in Tables III–VI.

3.2. Implementation of the θ projection technique

Application of the θ projection technique requires a specification for the functional forms of η and g_j in Equations 1a and b above. In the past a variety of different functional forms have been put forward [13, 14]. Traditionally the θ projection technique has used the following 4Θ expression because it has been shown to give a good representation (at least for large strains) of experimental creep curves at any test condition [15]

$$\varepsilon = \Theta_1(1 - e^{-\Theta_2 t}) + \Theta_3(e^{\Theta_4 t} - 1). \qquad (2a)$$

Very recently the fit of Equation 2a to an experimental creep curve at small strains has been shown to be quite poor. This miss specification has lead to considerable errors in the calculation of times to small strains and to initial and minimum creep rates in various steels [1, 9] and aluminium alloys [8]. So in recent years the theta projection technique has been modified by Evans [8] to allow the option of using the following 6θ modification to Equation 2a

$$\varepsilon = \theta_1(1 - e^{-\theta_2 t}) + \theta_3(e^{\theta_4 t} - 1) + \theta_5(1 - e^{-\theta_6 t}). \qquad (2b)$$

These two variants have become known as the 4Θ and 6θ theta projection techniques and values for all the theta parameters are estimated using the non linear least squares technique in Evans [15]. The advantage of both these forms is that theory suggests that each θ_j and each Θ_j are functionally related to both stress and temperature. Traditionally, the following representation for the function g_j in Equation 1b has been used when applying the theta projection technique

$$\ln(\Theta_j) = a_{j1} + a_{j2}\tau + a_{j3}T + a_{j4}\tau T \quad j = 1,4$$
$$\ln(\theta_j) = b_{j1} + b_{j2}\tau + b_{j3}T + b_{j4}\tau T \quad j = 1,6 \qquad (2c)$$

Whilst a lot of recent research as gone into improving the modelling of strain with respect to time (i.e., the 6θ model) little has been done to assess the validity of the theta interpolation/extrapolation function given by Equation 2c.

3.3. Possible theta interpolation/extrapolation functions

Indeed there are theoretical reasons for believing that Equation 2c may well be adequate for interpolation

TABLE IX Mean absolute percentage prediction errors made using the 4Θ prediction technique with the interpolation/extrapolation function given by Equation 4b

Temperature	Type of prediction	Unweighted (%)	Weighted (%)
783 K	Interpolation	17.45	16.52
	Extrapolation	132.45	73.08
	Overall	119.68	66.79
803 K	Interpolation	34.2	35.99
	Extrapolation	168.50	56.8
	Overall	87.92	44.32
823 K	Interpolation	31.46	26.18
	Extrapolation	427.72	39.19
	Overall	389.20	37.93
843 K	Interpolation	13.84	40.34
	Extrapolation	933.11	110.64
	Overall	508.83	78.19
863 K	Interpolation	38.28	41.46
	Extrapolation	27777.06	279.56
	Overall	2480.04	255.22
923 K	Extrapolation	238.99	197.06
Other	Extrapolation	634.04	543.94
	Overall	560.46	478.83
All temperatures	Interpolation	28.08	31.93
	Extrapolation	973.7	153.01
	Overall	**853.68**	**137.64**

The interpolation error is the mean absolute % prediction error made by the theta prediction technique using the interpolation/extrapolation function given by Equation 4b at the accelerated test conditions shown in Table II. The extrapolation error is the mean absolute % prediction error made by the theta prediction technique using the interpolation/extrapolation function given by Equation 4b at the NRIM test conditions shown in Table II. The overall error is the mean absolute % prediction error made by the theta prediction technique using the interpolation/extrapolation function given by Equation 4b at all the test conditions shown in Table II. Other temperatures are 723 K, 803 K, 898 K and 948 K. Unweighted refers to prediction errors made using the ordinary least squares estimates of the coefficients of Equation 4b shown in Tables III–VI. Weighted refers to prediction errors made using the weighted least squares estimates of the coefficients of Equation 4b shown in Tables III–VI.

TABLE X Mean absolute percentage prediction errors made using the 4Θ prediction technique with the interpolation/extrapolation function given by Equation 4d

Temperature	Type of prediction	Unweighted (%)	Weighted (%)
783 K	Interpolation	22.01	16.35
	Extrapolation	213.13	93.77
	Overall	191.89	85.17
803 K	Interpolation	34.81	35.85
	Extrapolation	228.48	54.49
	Overall	112.28	43.31
823 K	Interpolation	32.91	24.99
	Extrapolation	437.44	41.49
	Overall	398.11	39.89
843 K	Interpolation	15.63	40.21
	Extrapolation	714.29	57.59
	Overall	391.83	49.57
863 K	Interpolation	35.48	49.36
	Extrapolation	1656.16	44.53
	Overall	1480	45.31
923 K	Extrapolation	41.09	31.84
Other	Extrapolation	311.37	142.95
	Overall	277.84	128.15
All temperatures	Interpolation	28.99	33.32
	Extrapolation	650.25	55.67
	Overall	**571.39**	**52.84**

The interpolation error is the mean absolute % prediction error made by the theta prediction technique using the interpolation/extrapolation function given by Equation 4d at the accelerated test conditions shown in Table II. The extrapolation error is the mean absolute % prediction error made by the theta prediction technique using the interpolation/extrapolation function given by Equation 4d at the NRIM test conditions shown in Table II. The overall error is the mean absolute % prediction error made by the theta prediction technique using the interpolation/extrapolation function given by Equation 4d at all the test conditions shown in Table II. Other temperatures are 723 K, 803 K, 898 K and 948 K. Unweighted refers to prediction errors made using the ordinary least squares estimates of the coefficients of Equation 4d shown in Tables III–VI. Weighted refers to prediction errors made using the weighted least squares estimates of the coefficients of Equation 4d shown in Tables III–VI.

TABLE XI Mean absolute percentage prediction errors made using the 4Θ prediction technique with the interpolation/extrapolation function given by Equation 4e

Temperature	Type of prediction	Unweighted (%)	Weighted (%)
783 K	Interpolation	12.67	10.25
	Extrapolation	97.79	59.86
	Overall	88.34	54.35
803 K	Interpolation	35.47	36.34
	Extrapolation	161.51	40.89
	Overall	85.89	38.16
823 K	Interpolation	33.15	23.38
	Extrapolation	431.31	57.44
	Overall	392.60	54.13
843 K	Interpolation	12.97	35.77
	Extrapolation	910.19	164.59
	Overall	496.09	105.14
863 K	Interpolation	34.90	35.18
	Extrapolation	2905.30	317.35
	Overall	2589.40	289.05
923 K	Extrapolation	84.61	237.85
Other	Extrapolation	734.57	210.54
	Overall	648.98	185.47
All temperatures	Interpolation	27.06	27.7
	Extrapolation	979.09	163
	Overall	**858.25**	**145.82**

The interpolation error is the mean absolute % prediction error made by the theta prediction technique using the interpolation/extrapolation function given by Equation 4e at the accelerated test conditions shown in Table II. The extrapolation error is the mean absolute % prediction error made by the theta prediction technique using the interpolation/extrapolation function given by Equation 4e at the NRIM test conditions shown in Table II. The overall error is the mean absolute % prediction error made by the theta prediction technique using the interpolation/extrapolation function given by Equation 4e at all the test conditions shown in Table II. Other temperatures are 723 K, 803 K, 898 K and 948 K. Unweighted refers to prediction errors made using the ordinary least squares estimates of the coefficients of Equation 4e shown in Tables III–VI. Weighted refers to prediction errors made using the weighted least squares estimates of the coefficients of Equation 4e shown in Tables III–VI.

purposes at intermediate temperatures but that it is unlikely to be suitable for extrapolation purposes. An insight into why this might be the case can be seen by taking Kachanov's [16] damage parameter approach to failure. Here the creep rate is given a power law representation

$$\frac{d\varepsilon}{dt} = \dot{\varepsilon} = A\tau^m(1-\omega)^{-m} \approx A\tau^m(1+m\omega) \quad (3a)$$

where ω is the fraction of a specimens cross section occupied by voids and A and M is a model constants. If ω is assumed to be directly proportional to the strain in tertiary creep, ($\omega = C\varepsilon$), then

$$\dot{\varepsilon} = \dot{\varepsilon}_m(1 + mC\varepsilon) \quad (3b)$$

where $\dot{\varepsilon}_m$ is the minimum creep rate given by $A\tau^m$. The solution to Equation 3b is then

$$\varepsilon = \frac{1}{mC}(e^{mC\dot{\varepsilon}_m t} - 1) \quad (3c)$$

Comparing Equation 3c with Equation 2a gives $\Theta_3 = 1/mC$ and $\Theta_4 = mC\dot{\varepsilon}_m$. m and C and thus Θ_3 are geometrical factors that should not depend rapidly on stress or temperature. However, the minimum creep rate and thus Θ_4 is expected to vary with stress and temperature. A generalisation given by Evans [4] leads to a similar conclusion for Θ_1 and Θ_2. Thus Θ_2 and Θ_4 are often termed rate constants and Θ_1 and Θ_3 strain like quantities. The dependency of Θ_2 and Θ_4 on stress and temperature may be treated on an atomic scale by considering the jump rate of the dislocations

$$\text{Forward jump rate} = B\exp\left[-\frac{U - V\tau}{kT}\right],$$

$$\text{Backward jump rate} = B\exp\left[-\frac{U + V\tau}{kT}\right],$$

$$\text{Resultant jump rate} = 2B\exp\left[-\frac{U}{kT}\right]\sinh\left[\frac{V\tau}{kT}\right],$$

where U is the activation energy required to move a dislocation in the absence of an external stress, τ is the shear stress, k is the universal gas constant and V the activation volume. Thus under the assumption that the backward jump is small enough to ignore,

$$\ln[\Theta_j] = c_{j1} + c_{j2}\frac{1}{T} + c_{j3}\frac{\tau}{T}. \quad (4a)$$

Equation 4a clearly differs from Equation 2c. Over a narrow and intermediate temperature range the parabolic relation between $\ln[\Theta_j]$ and T is approximately linear so that Equation 2c can be seen as an approximation to Equation 4a over such a range. Hence Equation 2c may be adequate for interpolation with respect to temperature changes but is likely to be inferior to Equation 4a when it comes to extrapolation. Notice also that in Equation 4a, stress enters into the expression only via its interaction with temperature. A possible generalisation of Equation 4a would be to include stress on its own

$$\ln[\Theta_j] = d_{j1} + d_{j2}\frac{1}{T} + d_{j3}\frac{\tau}{T} + d_{j4}\tau. \quad (4b)$$

If the backward jump were also taken into account another generalisation of Equation 4a would be

$$\ln[\Theta_j] = e_{j1} + e_{j2}\frac{1}{T} + \sinh\left[e_{j3}\frac{\tau}{T}\right] \quad (4c)$$

However, the identification of this equation may require a wider range of test conditions than those typically used in accelerated testing. Further, Eyring *et al.* [17] have suggested that the above Arrhenius type expressions in temperature may lack theoretical justification when factors other than temperature are important in determining rate processes. An Eyring model based on chemical reaction rate theory and quantum mechanics for the minimum creep rate would suggest the following functional form for the theta interpolation/extrapolation function

$$\ln[\Theta_j] = f_{j1} + f_{j2}\frac{1}{T} + f_{j3}\frac{\tau}{T} + f_{j4}\ln\frac{1}{T} \quad (4d)$$

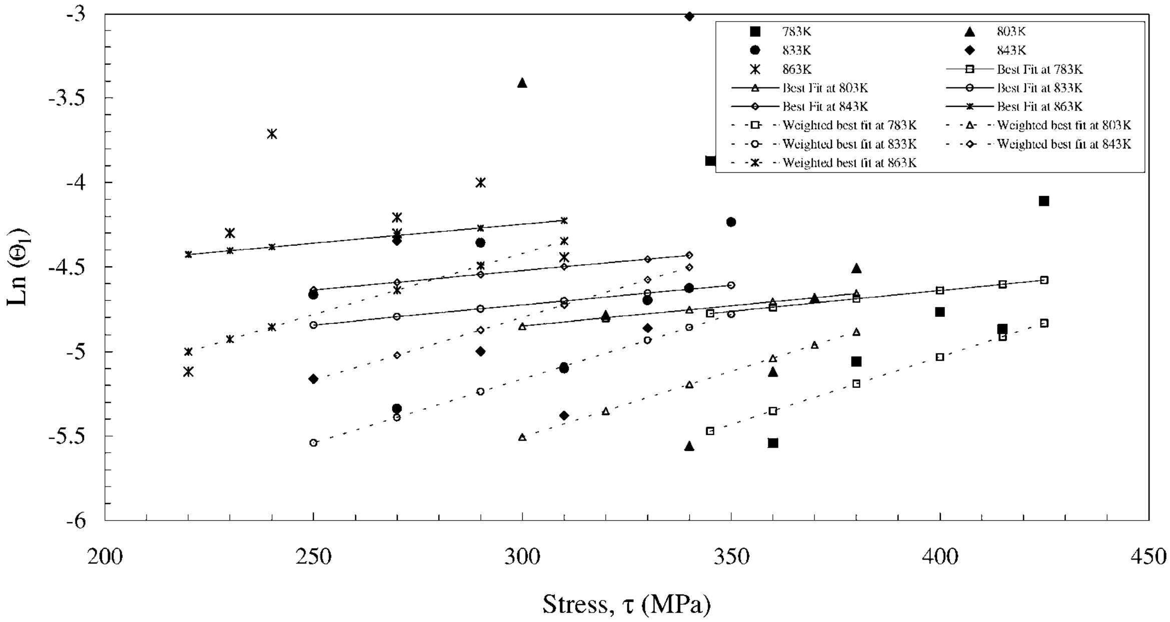
A possible generalisation of this would be to include stress on its own

$$\ln[\Theta_j] = h_{j1} + h_{j2}\frac{1}{T} + h_{j3}\frac{\tau}{T} + h_{j4}\ln\frac{1}{T} + h_{j5}\tau \quad (4e)$$

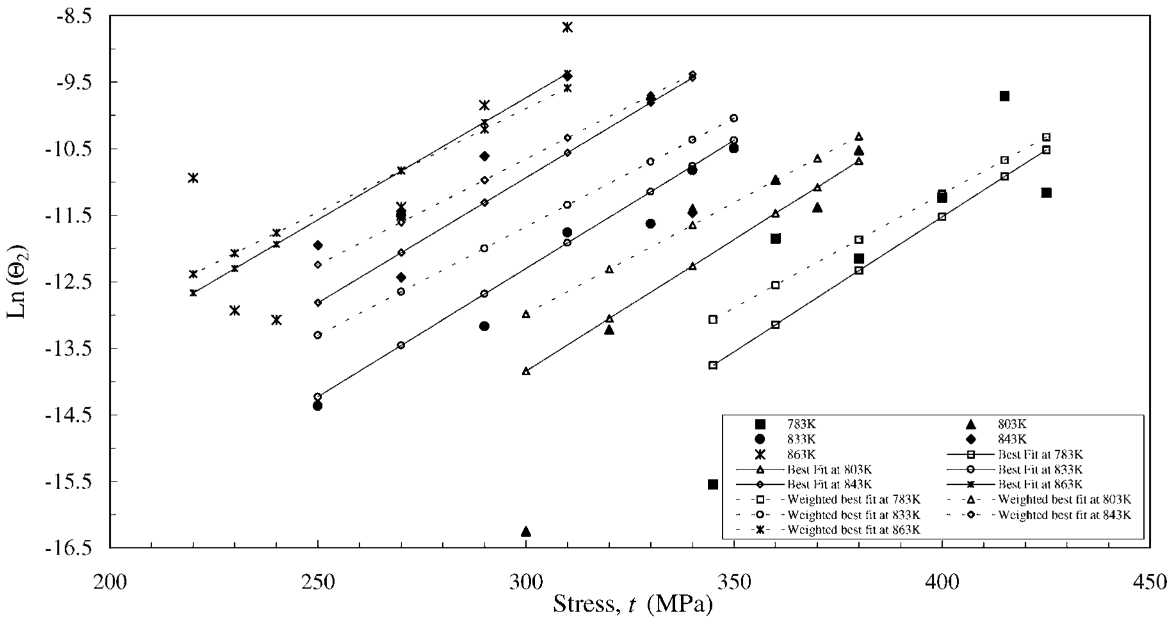
The parameters in Equations 4 can be estimated either by weighted or ordinary least squares. In ordinary least squares the parameter values in each of these equations (i.e. C_j to k_j values) are chosen so as to minimise the squared difference between each measured value for Θ_j and that predicted by the equation under consideration. In weighted least squares each variable (including the intercept terms) is first multiplied by a weight and least squares applied to these transformed equations. The weights reflect the fact that each Θ_j is not a known quantity but is instead estimated from the experimental data. So those Θ_j values with large variances should have small weights and those with small variances large weights according to the formula

$$W_j = \frac{\Theta_j^2}{\mathrm{Var}[\Theta_j]} \quad (4f)$$

where $\mathrm{Var}[\Theta_j]$ is the variance associated with the estimated value for Θ_j. Evans [10] has suggested a robust method for estimating such variances and this is the technique used in this paper. Section 4 below looks at how sensitive the 4Θ rupture time predictions are to these possible functional forms and to the procedure of weighting.

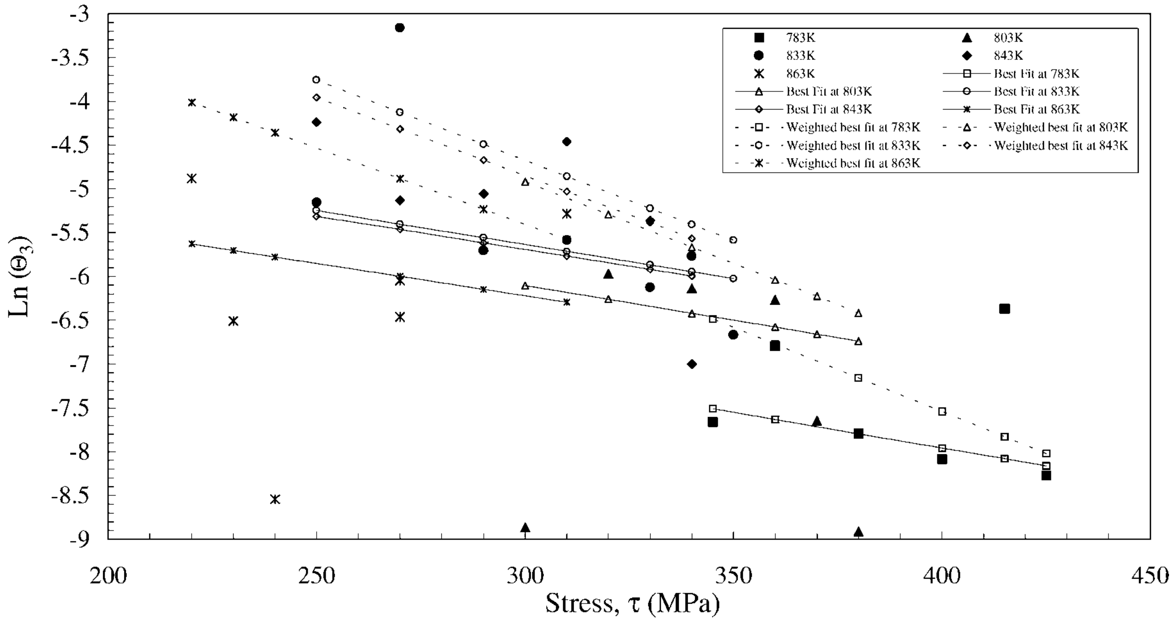


(a)



(b)

Figure 1 (a) Variation of Θ_1 with stress and temperature together with weighted and unweighted interpolationss. (b) Variation of Θ_2 with stress and temperature together with weighted and unweighted interpolations. (c) Variation of Θ_3 with stress and temperature together with weighted and unweighted interpolations. (d) Variation of Θ_4 with stress and temperature together with weighted and unweighted interpolations. (e) Variation of ε_R with stress and temperature together with weighted and unweighted interpolations. (*Continued*).



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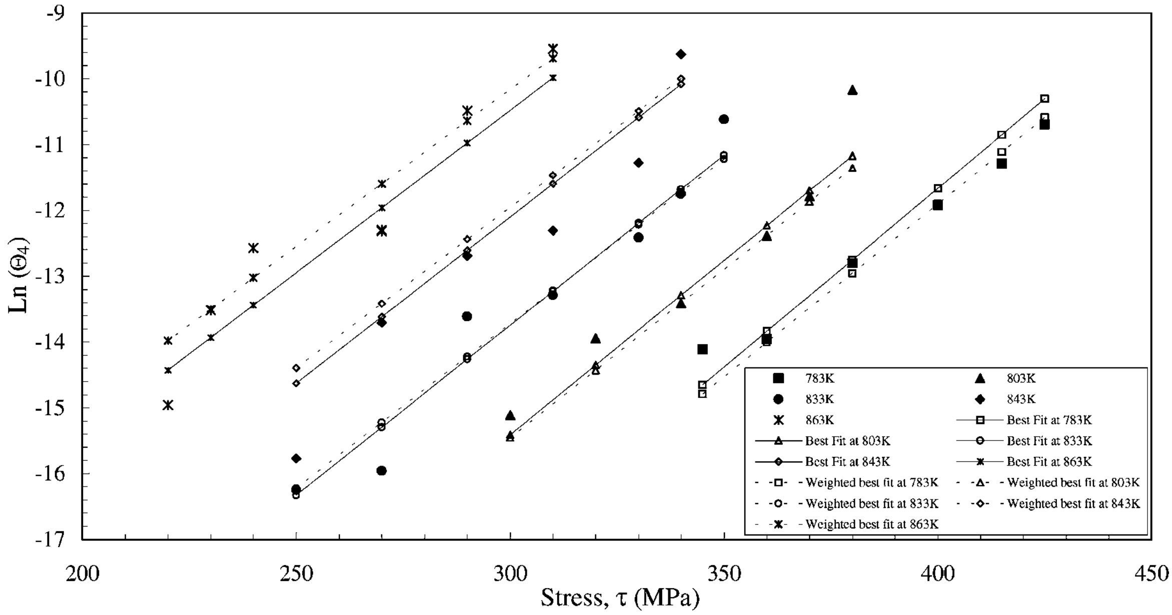

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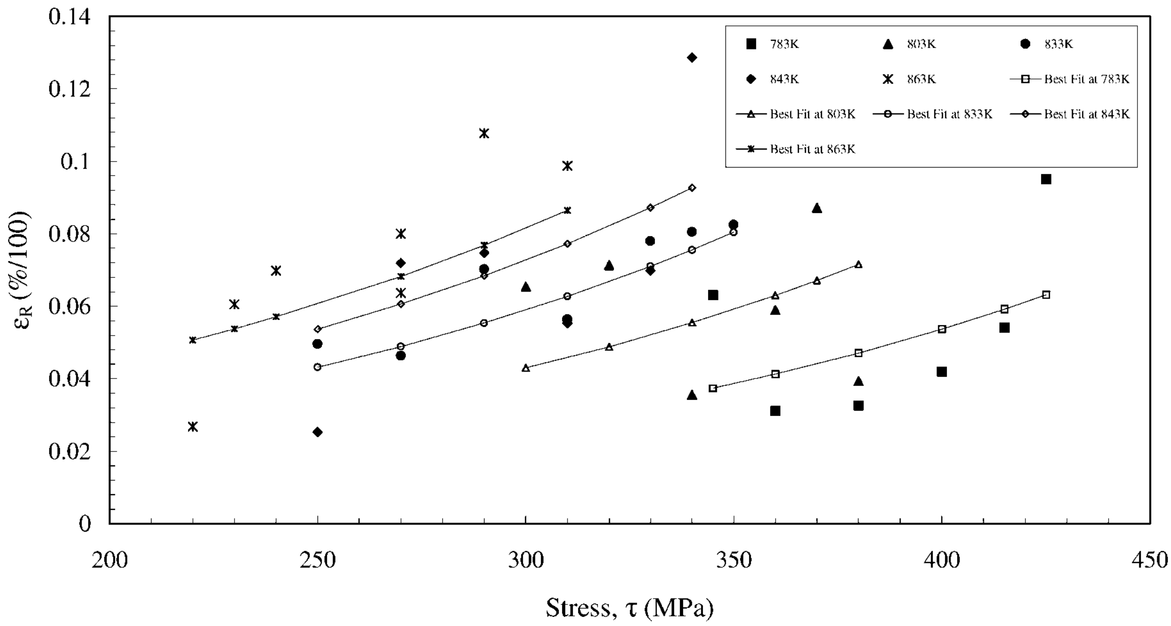


(e)

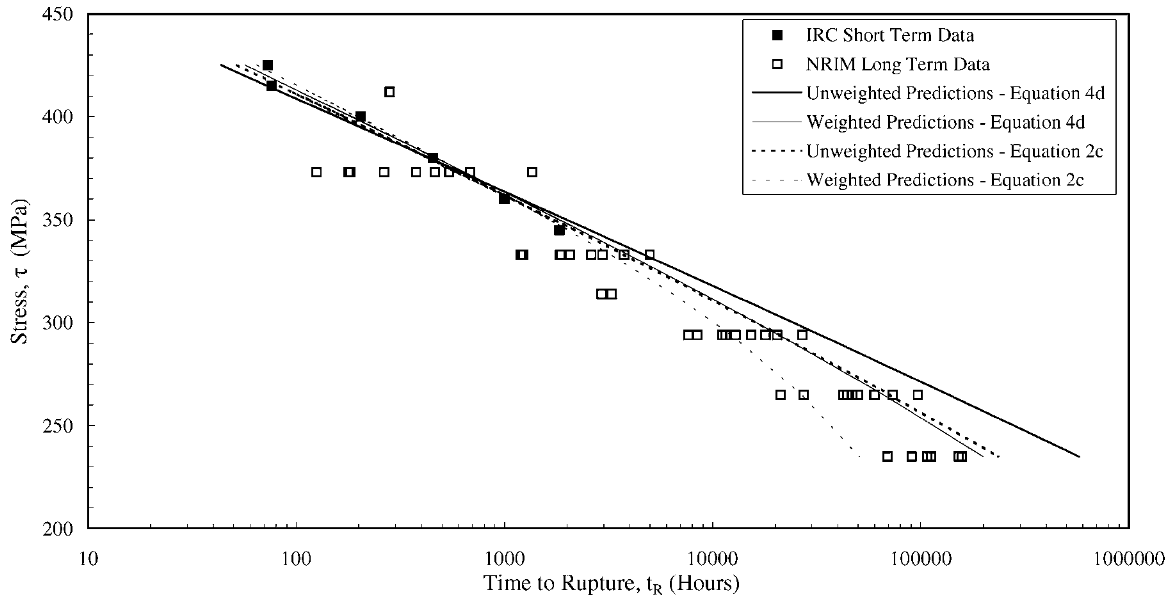
Figure 1 (*Continued*).

3.4. Assessing the accuracy of predictions made from the 4Θ technique

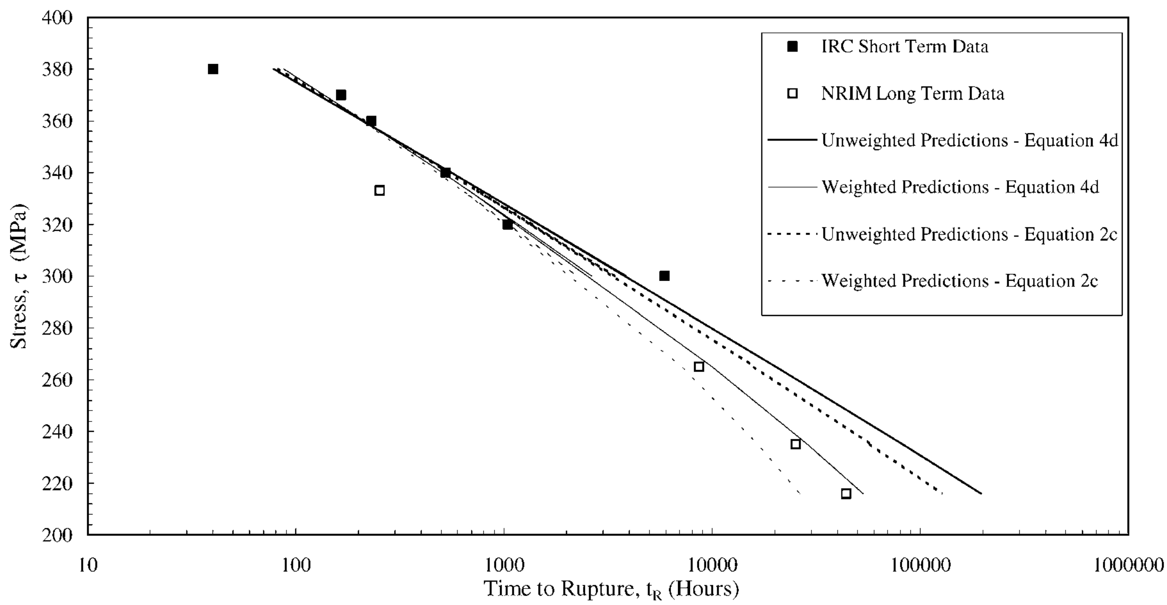
Any one of the above theta interpolation/extrapolation functions can be used to project the Θ_j values to any stress-temperature combination. These can then be inserted into Equation 2a and the rupture time predicted by finding that value for t that yields the predicted rupture strain, ε^P_{Ri}, at that ith test condition. The predicted rupture strain is in turn found by replacing $\mathrm{Ln}[\Theta_j]$ with $\mathrm{Ln}[\varepsilon]$ in Equation 2c and or 4a to e. The accuracy of such predictions over a range of test conditions can then be obtained using the following summary statistic

$$\mathrm{MAPE} = \sum_{i=1}^{N} \left[\frac{\frac{\left| t^A_{Ri} - t^P_{Ri} \right|}{t^A_{Ri}}}{N} \right], \tag{5}$$

where MAPE is the mean absolute percentage error, t^P_{Ri} is the predicted rupture time at the ith test condition, t^A_{Ri} the actual rupture time at the ith test condition and N the number of predictions made.

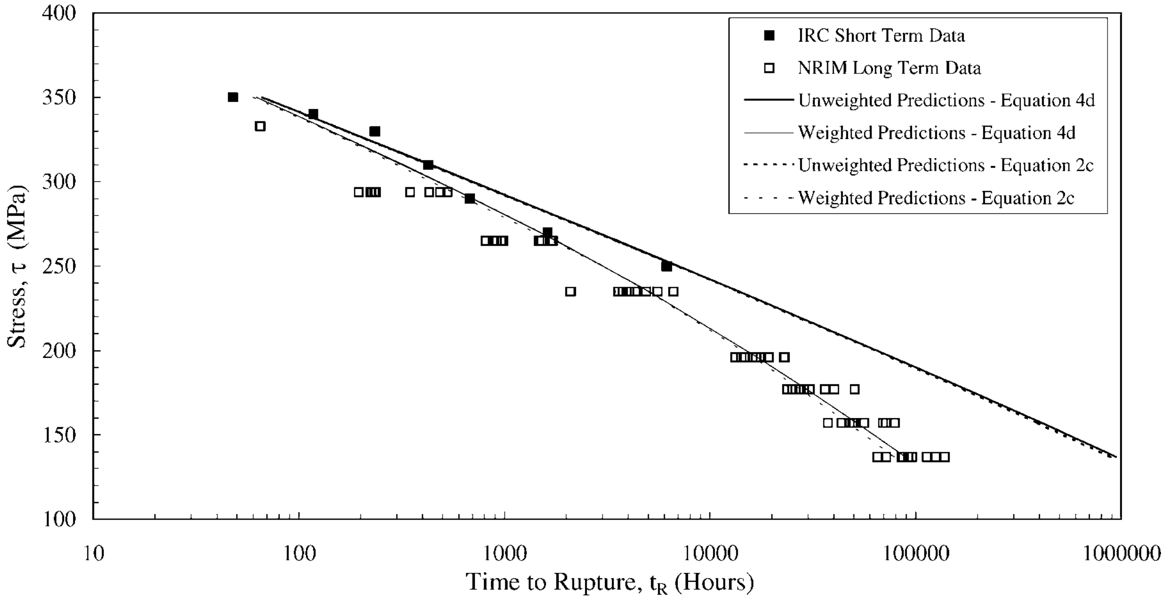


(a)

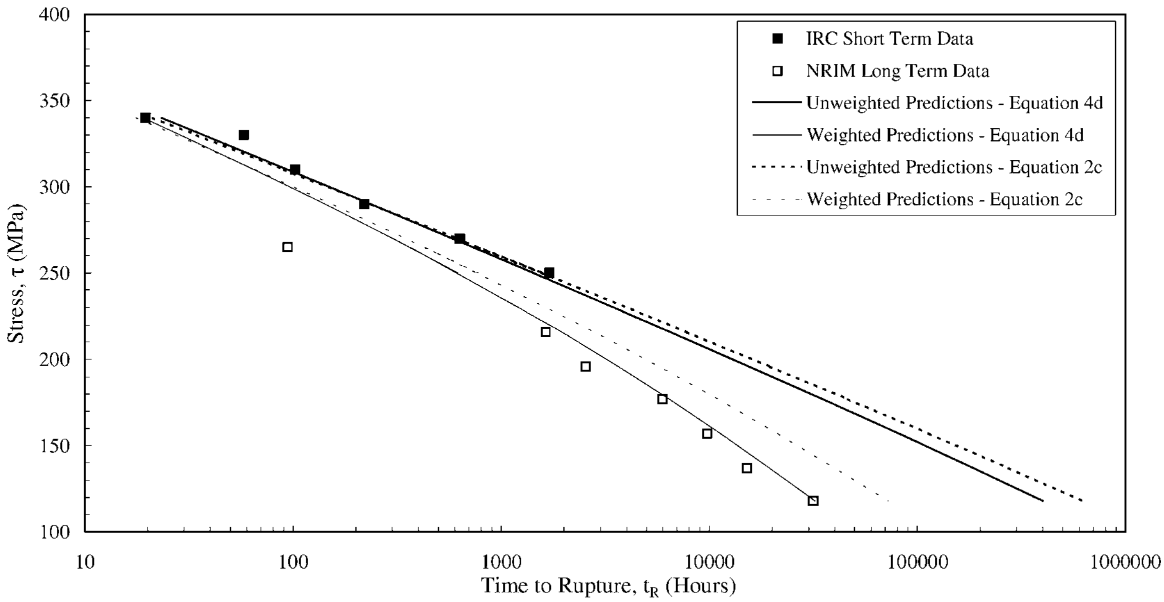


(b)

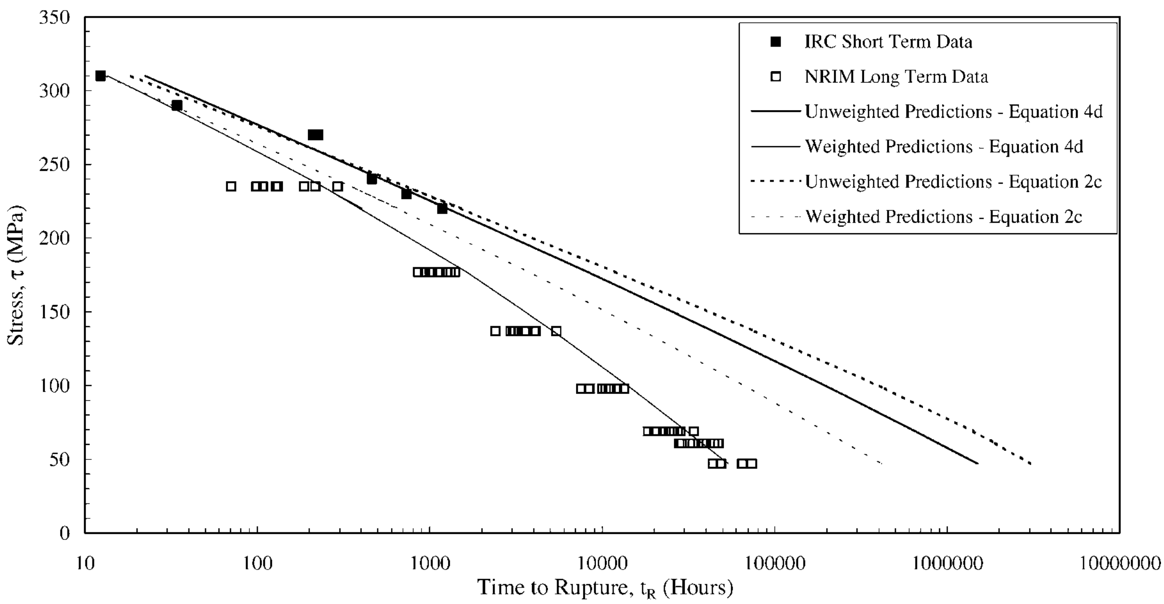
Figure 2 (a) Predicted σ/t_R plots for constant stress conditions, compared with measured t_R values obtained from short tests at 783 K (IRC) and long term tests at 783 K (NRIM). (b) Predicted σ/t_R plots for constant stress conditions, compared with measured t_R values obtained from short tests at 803 K (IRC) and long term tests at 803 K (NRIM). (c) Predicted σ/t_R plots for constant stress conditions, compared with measured t_R values obtained from short tests at 823 K (IRC) and long term tests at 823 K (NRIM). (d) Predicted σ/t_R plots for constant stress conditions, compared with measured t_R values obtained from short tests at 843 K (IRC) and long term tests at 843 K (NRIM). (e) Predicted σ/t_R plots for constant stress conditions, compared with measured t_R values obtained from short tests at 863 K (IRC) and long term tests at 863 K (NRIM). (f) Predicted σ/t_R plots for constant stress conditions, compared with measured t_R values obtained from short tests at 923 K (IRC) and long term tests at 923 K (NRIM). (g) Predicted σ/t_R plots for constant stress conditions, compared with measured t_R values obtained from long term tests at various other temperatures (NRIM). (*Continued.*)



(c)

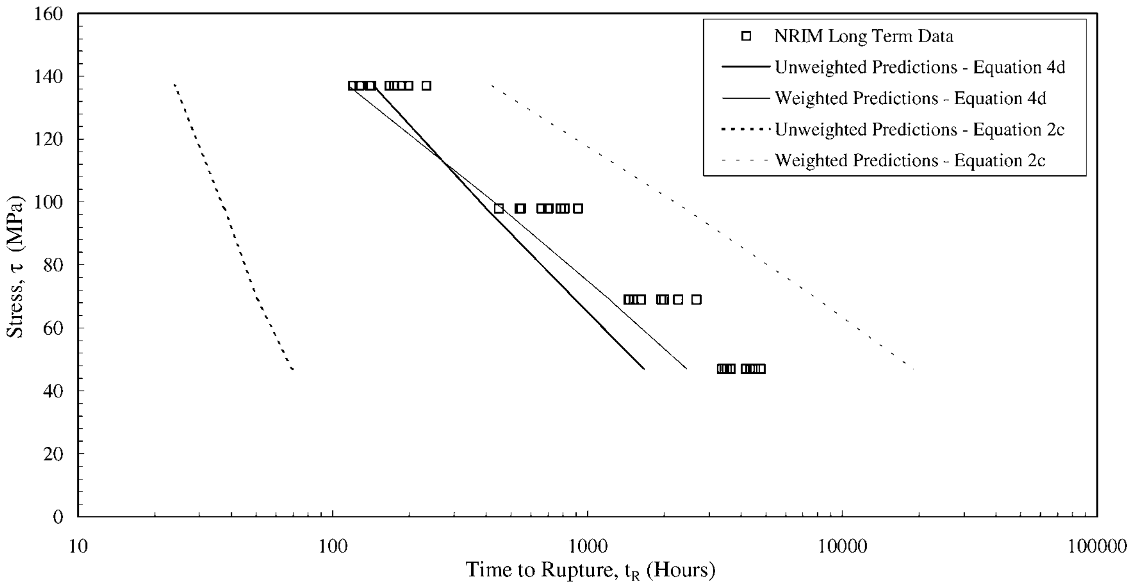


(d)

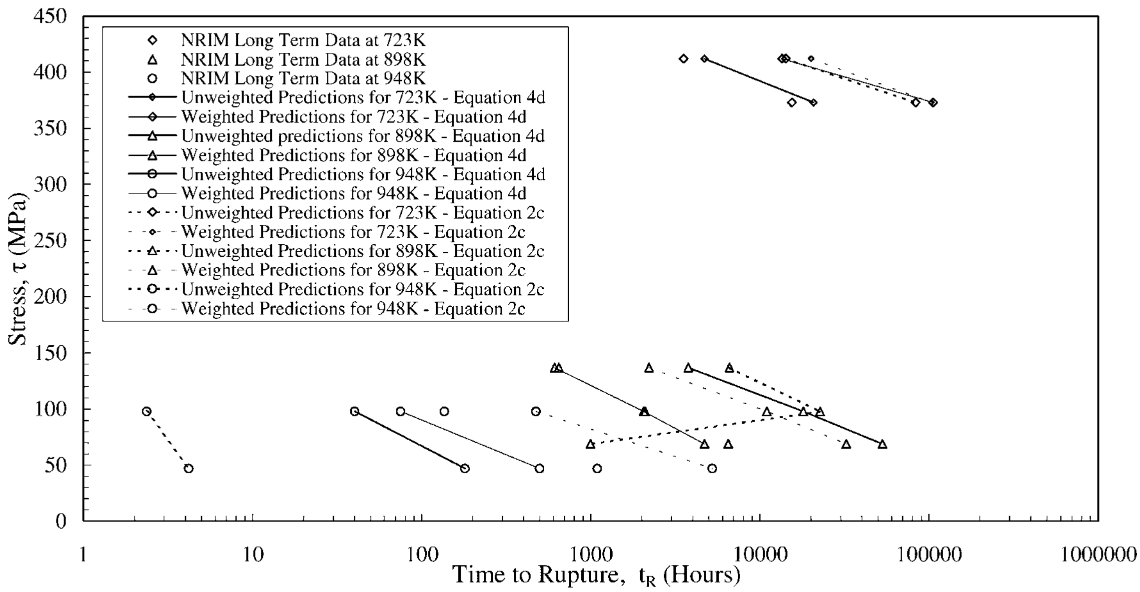


(e)

Figure 2 (*Continued.*)



(f)



(g)

Figure 2 (*Continued.*)

A MAPE can be calculated for any of the above theta interpolation/extrapolation functions. Further, a MAPE can also be calculated for just the accelerated (IRC) test conditions shown in Table II. This will be called an interpolation MAPE because each of the theta interpolation/extrapolation function shown above are estimated using the results from such test conditions. A MAPE can also be calculated for just the NRIM test conditions shown in Table II. This will be called an extrapolation MAPE because each of the theta interpolation/extrapolation function shown above are not estimated using results from such test conditions.

4. Results of sensitivity analysis

4.1. The interpolation/extrapolation functions for Θ_j

Fig. 1 show the experimental values for Θ_j at the accelerated test conditions together with the values predicted by Equation 4d using the estimated coefficients shown in Tables III to VI. Both the weighted and unweighted predictions are shown and as can be seen the weighting procedure does make a considerable difference to the predictions obtained. As expected the variation of Θ_1 and Θ_3 with stress and temperature is not as large as that for Θ_2 and Θ_4, although weighting the Θ_1 and Θ_3 estimates does lead to a more pronounced trend with respect to stress. The effect of weighting Θ_2 is a little more complicated. At low temperatures the fitted lines become shallower following weighting and this effect becomes more pronounced as the temperature is increased. For Θ_4, weighting seems to shift the fitted line down at low temperatures and then as the temperature increases this effect is diminished until at high temperatures weighting appears to shift the fitted line upwards. Fig. 1e shows the variation of rupture strain with stress and temperature, together with the interpolations given by Equation 4d (with $\ln[\varepsilon_R]$ replacing $\ln[\Theta_j]$).

Tables III to VI also show the results of estimating the other theta interpolation/extrapolation functions for Θ_j derived in Section 3.3 above. Both the ordinary least squares and weighted least squares estimates are given. It can be seen from Tables IV and VI, that only in Equation 4a are all the explanatory variables statistically significant when using ordinary least squares as the estimation technique. This may suggest that the more parsimonious the equation the better suited it will be to rupture time predictions.

Notice that Equation 4c is not estimated at all. This is because the value for e_{j3} in Equations 4c and c_{j3} in Equation 4a work out to be very similar, implying that the stress/temperature range for the accelerated test conditions in Table II are not broad enough to pick up any sinh non linearity. It may be the case that the sinh expression is more appropriate but more test conditions would be required to prove this. This could form an interesting area for future research.

4.2. The MAPE statistics

It is interesting to measure what impact the different theta interpolation/extrapolation functions, and their estimation via weighted or ordinary least squares, has on the resulting rupture time predictions. This impact is measured in Tables VII to XI using the mean absolute percentage error (MAPE) given by Equation 5 above. The predictions are also summarised in Fig. 2a to g where predictions are shown together with the shorter term IRC rupture data and the longer-term NRIM rupture data. The scatter in the NRIM data reflects the fact that each data point at a given stress corresponds to a different batch of material.

A number of important observations can be drawn from Tables VII to XI. First, and irrespective of the type of theta interpolation/extrapolation function used, the prediction is always better if such functions are estimated using weighted least squares. The overall prediction error is always much lower when a weighting scheme is used because such a weighting procedure seems to dramatically improve the predictions made in extrapolation without significantly worsening the predictions made in interpolation.

Secondly, the best overall predictions are obtained using the theta interpolation/extrapolation function given by either Equation 4d or 4a. If weighted least squares is used, then Equation 4d is best, if ordinary least squares is used then Equation 4a is best. This reflects the fact that the variable $\ln(1/T)$ often shows up as being more statistically significant, in Tables III–VI, when weighted least squares is used. It therefore appears to be the case that stress should only be included in the theta interpolation/extrapolation function as an interaction with temperature, i.e, τ/T should appear but not τ as well. Irrespective of weighting, the traditional theta interpolation/extrapolation function, given by Equation 2c, always produced the worst over all prediction error.

If predictions are then broken down by temperature some additional results stand out. At 783 K the IRC and NRIM data sets are broadly comparable at the higher stresses. Here Equation 2c actually extrapolates better than Equation 4d irrespective of the estimation technique as shown in Fig. 2a and Tables VII–XI. However this appears to be the exception rather than the rule.

At 803 K the IRC specimens appear to have had much longer lives than the NRIM specimens at the higher stresses. This is likely to reflect the higher nickel content present in the IRC test specimens. Despite this, all the theta interpolation/extrapolation functions produced similar interpolation errors but Equation 4d, as estimated using weighted least squares, produced marginally better extrapolations when compared to Equation 2c, as shown in Fig. 2b and Tables VII–XI.

At 823 K the IRC specimens appear to have produced very similar lives to the NRIM specimens at the higher stresses. Here Equations 4d and 2c produce almost identical interpolative and extrapolative errors in prediction. However, these errors are always lower if a weighted least squares procedure is used as shown in Fig. 2c and Tables VII–XI.

At 843 K the IRC specimens appear to have had much longer lives than the NRIM specimens at the higher stresses. Again this is likely to reflect the higher nickel content present in the IRC test specimens Despite this all the theta interpolation/extrapolation function produced similar interpolation errors but Equation 4d as, estimated using weighted least squares, produced much better extrapolations when compared to Equation 2c, as shown in Fig. 2d and Tables VII–XI. The same is true if ordinary least squares is used instead.

At 863 K the IRC specimens appear to have had much longer lives than the NRIM specimens at the higher stresses. Despite this each theta interpolation/extrapolation function produced similar interpolation errors. Notice that the weighting scheme placed most emphasis on the failure times obtained at the higher two stresses so that the interpolative predictions appear to be too small over all. However, as a result of this emphasis, the extrapolative predictions are much better and are indeed excellent when using the theta interpolative/extrapolative function given by Equation 4d, as shown in Fig. 2e and Tables VII–XI.

At 923 K all the rupture time predictions are extrapolative with respective to temperature as no IRC data was available at this temperature. All the previous figures have shown extrapolations with respect to stress as all the temperatures in these figures are IRC test temperatures. In Fig. 2f, the extrapolations with respect to temperature are only reasonable if Equation 4d is used as the theta interpolation/extrapolation function. Further the extrapolations are better if Equation 4d is estimated using weighted least squares.

Again in Fig. 2g all the predictions are extrapolative with respect to temperature as no IRC data are available at the temperatures shown. Again the predictions are much better when Equation 4d is used as the theta interpolation/extrapolation function. Again it is best to estimate this function using weighted least squares.

5. Conclusions

A number of conclusions can be drawn from the results above. First, irrespective of the functional form chosen for the theta interpolation/extrapolation function, it is always best to estimate such a function using weighted least squares. In particular the extrapolated rupture times are always more accurate using such weighted estimates. Secondly, the theta interpolation/extrapolation function traditionally used in the theta projection technique turns out to produce some of the most inaccurate rupture time extrapolations. Third, the theta interpolation/extrapolation function that produces the most reliable forecast over a range of test conditions includes only the terms $1/t$ and t/T. Indeed when extrapolating with respect to temperature the use of the terms T and tT in the traditional function produce very poor rupture time predictions. At some temperatures, predictions are marginally improved by including the additional term $\ln(1/T)$.

It would be interesting to see if the interpolation/extrapolation function that has been shown to be best for 1CrMoV rotor steel is also the best for other steels as well. Further, it may beneficial to estimate creep curves over a wide range of test conditions to see if the sinh term can further improve rupture time predictions using the theta technique. These could form areas for future research.

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